Mathematical Induction

Applications

1. If there are n lines on the plane such that no two of them are parallel and no three of them concurrent, find the total number of segments. In how many sections do these lines partition the plane?

2. There are n circles on the plane. Show that two colours is enough to shade the regions divided by the circles so that no two adjacent regions is of the same colour.

3. How many triangles can a convex n-sided polygon be divided by non-intersecting diagonals. How about the case when the polygon is not necessarily convex?

4. **(i)** Find the length of the sides of the regular 2^n – sided polygon inscribed in a circle of radius r .

(ii) By using the length of the circumference of the circle, find

$$\lim_{n\to\infty} 2^n \underbrace{\sqrt{2-\sqrt{2+\sqrt{2-....+(-1)^n\sqrt{2}}}}}_{n-1 \text{ times}}$$

(iii) By considering the 96 – sided regular polygon, find an approximation for π (Lin Hui).

(iv) Let S_n be the area of the regular n-sided polygon inscribed in the circle of radius r .

Show that: $\dfrac{S_{2^n}}{S_{2^{n+1}}} = \cos\dfrac{180^\circ}{2^n}$.

(v) By using the fact that $\lim\limits_{n\to\infty} S_{2^n} = \pi r^2$, show that $\dfrac{2}{\pi} = \cos 45^\circ \cos\dfrac{45^\circ}{2}\cos\dfrac{45^\circ}{4}....$

(vi) By using the fact that $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$, show the Vieta formula:

$$\pi = \frac{2}{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\left(1+\sqrt{\frac{1}{2}}\right)}\sqrt{\frac{1}{2}\left(1+\sqrt{\frac{1}{2}\left(1+\sqrt{\frac{1}{2}}\right)}\right)}....}$$

5. Generalize the result : $X \setminus (A_1 \cap A_2) = (X \setminus A_1) \cup (X \setminus A_2)$ (de Morgan's Law)

6. Find $\dfrac{d^n}{dx^n}\left(\dfrac{1}{1+x}\right)$.